AN APPROXIMATE METHOD OF CALCULATING THE KINETICS OF THE DRYING PROCESS

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An approximate method is presented for the calculation of the kinetics involved in the process of drying moist materials, this method based on the determination of the newly introduced Rebinder number and on the drying-rate relationships.

The kinetics of the process involved in the drying of moist materials is described by the variation with time in the mean integral values of the moisture content $\bar{u}$ and the temperature $\bar{t}$ of the material. These values can be derived by solving the differential equations for the transfer of moisture and heat in capillary-porous colloidal materials. However, to use these solutions we must know the transport coefficients. Moreover, the coefficients of moisture and heat transfer vary significantly as a result of the moisture content and temperature of the material so that, strictly speaking, the system of differential equations for moisture and heat transfer is a nonlinear system whose solution involves great difficulties [4]. Thus simplified methods of calculation have become an urgent necessity. Since the changes in the mean integral characteristics ($\bar{u}$ and $\bar{t}$) of the material must be known to describe the kinetics of the drying process, it would seem that we could employ the quantitative relationships governing external moisture and heat transfer, i. e., the relationships for the intensities of moisture transfer (j) and heat transfer (q). However, the calculation of j and q according to the Dalton and Newton formulas for a period of a diminishing drying rate is impossible, since the coefficients of moisture and heat transfer vary with time since the drying process is a typical nonsteady process of moisture and heat transfer. In first approximation the intensity of moisture transfer during a period of diminishing rates can be defined by the following formula proposed by one of the authors of [1,3]:

$$j = \left(\frac{d\bar{u}}{d\tau}\right) R_v \rho_0 = \rho_0 R_v K (\bar{u} - u_p) = \rho_0 R_v \frac{\varkappa N}{100} (\bar{u} - u_p), \tag{1}$$

where the drying coefficient K is directly proportional to the drying rates in a period of a constant rate N (%/hr). The relative drying coefficient $\varkappa$ in first approximation can be calculated from the relationship $\varkappa = 1.8/\bar{u}_0$, where $\bar{u}_0$ is the initial mean moisture content of the material.

V. V. Krasnikov further developed this method [2], based on the fact that the drying-rate curve during a period of decline is replaced by a broken straight line, i. e., the period of the declining drying rate is divided into two zones in each of which the drying rate is diminished as a function of the moisture content in linear fashion. Here we must know the values of the two relative drying coefficients $\varkappa_1$ and $\varkappa_2$.

The intensity of the moisture transfer in the drying process at the present time may thus be calculated rather exactly for various materials.

The intensities of heat and moisture transfer in the drying process can be related using the law of conservation of energy, thus reducing the calculation of the heat transfer to the calculation of the moisture transfer, and vice versa. This relationship has the form

$$q(\tau) = j(\tau)\left(r + c\frac{d\bar{t}}{d\bar{u}}\right) = \rho_0 R_v r\left(\frac{d\bar{u}}{d\tau}\right)\left[1 + \frac{c}{r}\frac{d\bar{t}}{d\bar{u}}\right], \tag{2}$$

where c is the reduced specific heat capacity of the moist material

$$c = c_0 + c_l \bar{u}. \tag{3}$$

The quantity $d\bar{t}/d\bar{u}$ characterizes the rise in the mean temperature of the material with a change in its average moisture content and is the thermal diffusivity b of the drying process ($b = d\bar{t}/d\bar{u}$); the generalized variable bc/r is a dimensionless number which serves as the basic characteristic for the kinetics of the process and is numerically equal to the ratio of the quantity of heat expended on the heating of the material to the quantity of heat expended on the evaporation of the moisture during an infinitely small interval of time. We will refer to this generalized number as the Rebinder number in honor of the outstanding scientist, Academician P. A. Rebinder, the founder of the discipline on the forms of the relationship of moisture and colloidal capillary-porous materials (we will denote the Rebinder number as Rb to distinguish it from Re used to denote the Reynolds number)

$$\mathrm{Rb} = \frac{bc}{r} = \frac{c}{r}\left(\frac{d\bar{t}}{d\bar{u}}\right). \tag{4}$$

The Rebinder number is a function of the thermal diffusivity b of the drying process, of the specific heat capacity c of the material, and of the specific heat of evaporation r for the moisture, while the quantities c and r are functions of the forms of the relationship between the moisture and the moist material; the heat of evaporation contains within itself not only the heat of evaporation r_l for the liquid, but the heat of wetting r_w as well ($r = r_l + r_w$).

Equation (2) in criterial form may be written as follows:

$$\mathrm{Ki}_q(\tau) = \mathrm{Ki}_m(\tau)\,\mathrm{Lu}\,\mathrm{Ko}\,(1 + \mathrm{Rb}), \quad (5)$$

where the Kirpichev numbers for heat and moisture transfer are

$$\mathrm{Ki}_q(\tau) = \frac{q(\tau) R_v}{\lambda T_\mathrm{m}}; \quad \mathrm{Ki}_m(\tau) = \frac{j(\tau) R_v}{a_m \rho_0 \bar{u}_0}. \quad (6)$$

The Lu number for the relative moisture in heat transfer is equal to ($\mathrm{Lu} = a_\mathrm{m}/a$), where a_m is the diffusion coefficient for moisture content and a is the coefficient of heat diffusion. The Kossovich number

$$\mathrm{Ko} = r\bar{u}_0/cT_\mathrm{m} \quad (7)$$

differs from the Rebinder number in that the latter characterizes the ratio of the local values of the heat expended on the heating and evaporation of the moisture, while the Ko number—a given quantity—is equal to the ratio of the heat expended on the evaporation of the entire moisture in the material to the heat of raising the temperature of the material from 0 to T_m. The following relationship exists between the Kossovich and the Rebinder numbers:

$$\mathrm{Ko} = B/\mathrm{Rb}, \quad (8)$$

where B is the dimensionless coefficient of thermal diffusivity for the drying process

$$B = b\frac{\bar{u}_0}{T_\mathrm{m}} = \frac{\partial(\bar{t}/T_\mathrm{m})}{\partial(\bar{u}/\bar{u}_0)}. \quad (9)$$

During the period of a constant drying rate the Rebinder number is equal to zero (Rb = 0), so that we have

$$q_\mathrm{c} = \rho_0 R_v \frac{N}{100} = \mathrm{const}.$$

We denote the ratio of the heat flux $q(\tau)$ during a period of a declining drying rate to the heat flux q_c during a period of a constant drying rate by $q^*(\tau)$ so that we have

$$q^*(\tau) = \frac{q(\tau)}{q_\mathrm{c}} = \left(\frac{d\bar{u}}{d\tau}\right)^* (1 + \mathrm{Rb}), \quad (10)$$

where $(d\bar{u}/d\tau)^* = (100/N)(d\bar{u}/d\tau)$ is the relative drying rate.

The heat transfer q^* and the moisture transfer $(d\bar{u}/d\tau)^*$ are related through the Rb number by Eq. (10), which is the basic equation for the kinetics of the drying process.

The change in the intensity of moisture transfer is defined by the approximate formula (1). To calculate the intensity of heat transfer it is necessary to know the relationship between the Rb number and the moisture content of the material ($\mathrm{Rb} = f(\bar{u})$). It should be borne in mind in this case that the Rebinder number can be calculated from the magnitude of the specific heat of the material ($\mathrm{Rb} = cb/r$) as well as from the magnitude of the specific heat of the dry body ($\mathrm{Rb}_0 = c_0 b/r$). The following relationship exists between these values of the Rebinder numbers:

$$\mathrm{Rb} = \mathrm{Rb}_0\left(1 + \frac{c_l}{c_0}\bar{u}\right). \quad (11)$$

The dimensionless coefficient of thermal diffusivity for the drying process

$$B = \mathrm{Rb}\,\mathrm{Ko} \quad (12)$$

makes it possible to establish the relationship between the mean temperature $\bar{t}$ of the material and its moisture content $\bar{u}$

$$\frac{\bar{t}}{T_\mathrm{m}} = -\int_{u_1}^{u_2} \frac{B(\bar{u})}{\bar{u}_0}\, d\bar{u}. \quad (13)$$

Thus, the finding of empirical formulas $\mathrm{Rb} = f(\bar{u})$ and $B = f(\bar{u})$ is of great interest not only to calculate the kinetics of the drying process, but also for the technology of drying, since the fundamental technological properties of the material subjected to drying are determined by its temperature and moisture content.

To ascertain the effect of the regime parameters of the drying process and the kind of material on the Rb number, as well as the effect on the latter of the dimensionless coefficient of thermal diffusivity B of the drying process during a period of a declining drying rate, we carried out experiments with various materials over a wide range of variations in temperature, drying rate, and relative humidity of the air.

The effect of the relative humidity of the air in the case of convective drying on the Rb number and on the coefficient B was studied on a ceramic plate 80 × 40 × × 6.5 mm in size at a temperature of $t_\mathrm{m} = 70°$ C for a variation in $\varphi = 20-80\%$. As we can see from Fig. 1, the relative humidity φ of the air in the investigated range exerts no effect on the magnitude of the Rb number. This is quite proper, since $\bar{t} = f(\bar{u})$ is independent φ and, consequently, the tangent to these curves with a specific value of $\bar{u}$ is a constant quantity.

The effect of the velocity of the heat carrier was investigated on cloth and a peat slab over a wide range of variations in the drying rate. As we can see from Fig. 2, the experimental points line up quite satisfactorily along a single curve $\bar{t} = f(w)$. Consequently, to a certain value of mean moisture content for the given material, it corresponds to a determined average value of its temperature, regardless of the air velocity. The coefficient of thermal diffusivity B and, consequently, the Rb_0 number are thus independent of the velocity of air.

It should be noted here that the $\bar{t} = f(w)$ curves will naturally be different for various forms of the materials, a fact which is associated with the form and nature of the relationship between the moisture and the material. However, the nature of the interrelationship between the average moisture content of the material and its temperature is significant.

Experiments show that the effect of the air temperature on the Rb number is different for capillary-porous materials (cloth and ceramics) and colloidal capillary-porous materials (peat slabs). It follows from

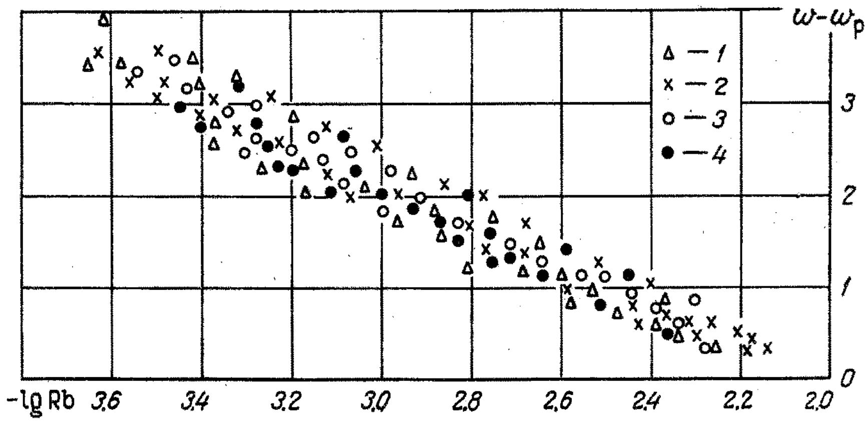

Fig. 1. Influence of relative humidity φ of the medium on the Rebinder number Rb with drying of a ceramic plate (T_m = = 343° K): 1) φ = 80%; 2) 60%; 3) 40%; 4) 20%.

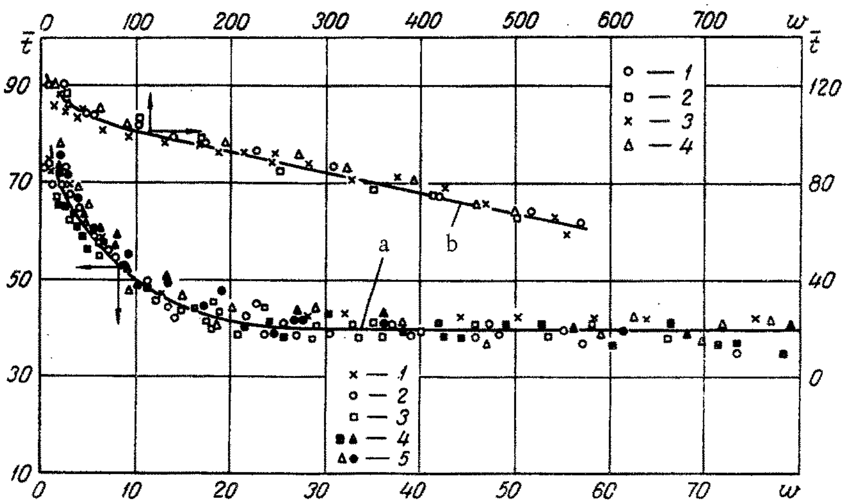

Fig. 2. Relation between mean material temperature $\bar{t}$ and mean moisture content of material w: a) cloth (T_m = 348° K and φ = 5%): 1) v = 0.0 m/sec; 2) 3.8; 3) 4.4; 4) 8.35; 5) 11.6; b) peat plate (T_m = 423° K and φ = 4%): 1) v = 1.2 m/sec; 2) 2.5; 3) 4.0; 4) 5.

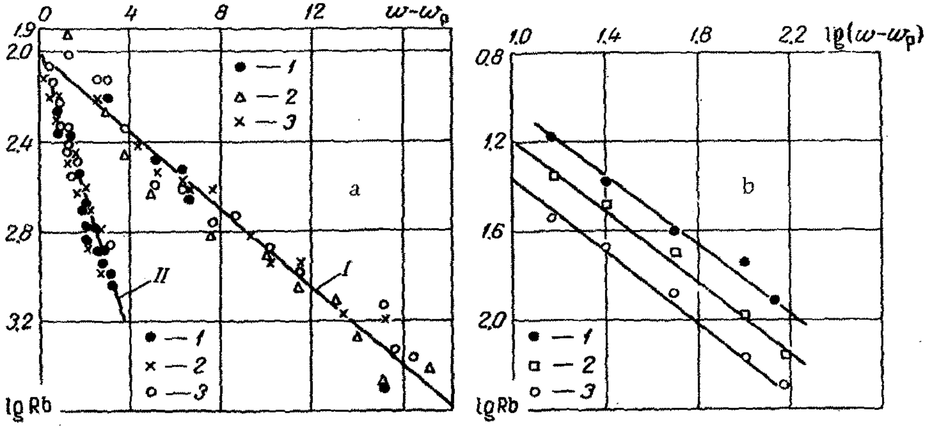

Fig. 3. Influence of temperature of the Rebinder number (φ = 5%): a) v = = 0.0 m/sec; I—ceramic plate: 1) T_m = 348° K; 2) 368; 3) 393; II—cloth: 1) T_m = 393° K; 2) 368; 3) 348; b) peat plate, v = 4.0 m/sec: 1) T_m = 473° K; 2) 423; 3) 373.

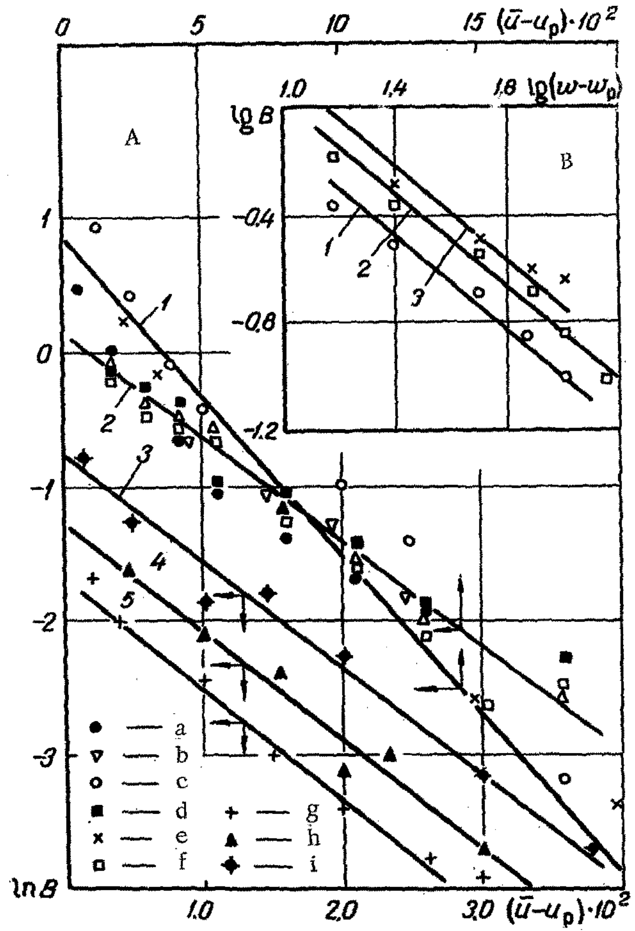

Fig. 4. Drying temperature factor B as function of mean moisture content of material ($\bar{u} - u_p$): A) 1) cloth, influence of medium temperature on coefficient B: a) T_m = 338° K; b) 368; c) 393; 2) cloth, influence of velocity of heat carrier on coefficient B: d) v = 11.6 m/sec; e) 8.35; f) 4.4; 3, 4, 5) ceramic plate, influence of temperature on coefficient B: g) T_m = 348° K; h) 363; i) 393; B) peat plate, influence of temperature of the medium on coefficient B: 1) 373° K; 2) 423; 3) 473.

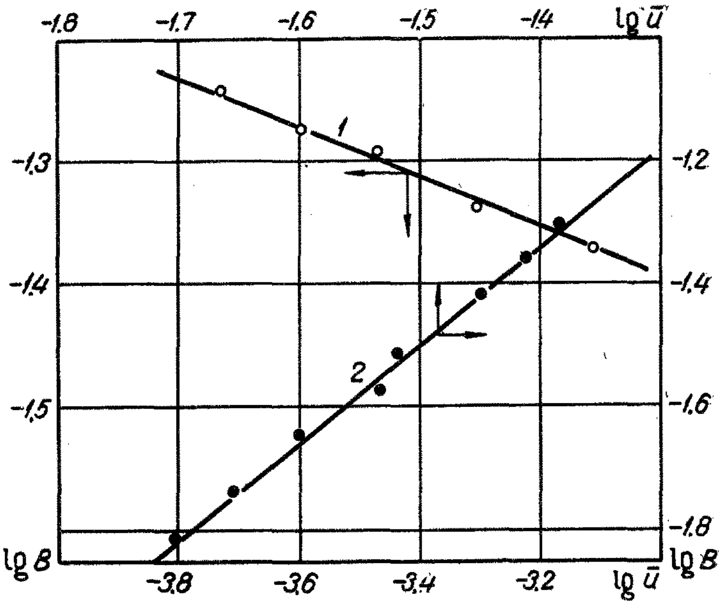

Fig. 5. Influence of drying parameters on drying temperature factor B: 1) polystyrene; 2) capron.

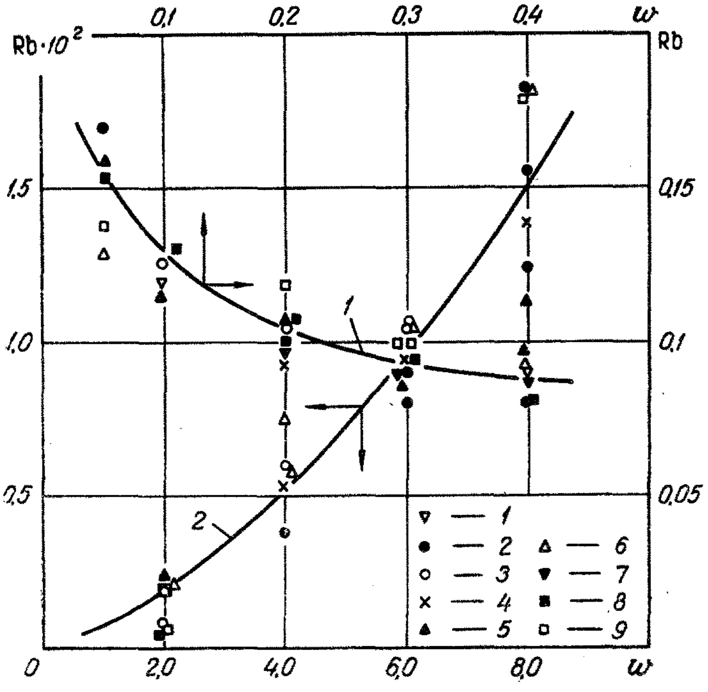

Fig. 6. Dependence of the Rebinder number Rb on the drying parameters: a) granulated polystyrene; b) granulated capron. 1–3) δ = 18, 12, 6 mm, T_{rad} = 723° K; v = 0.3 m/sec; 4–6) δ = 18, 12, 6 mm, T_{rad} = 823° K; v = 1.6 m/sec; 7–9) δ = 18, 12, 6 mm, T_{rad} = 953° K, v = 2.8 m/sec.

Fig. 3 that for cloth (Fig. 3a, curve I) and for ceramic (Fig. 3a, curve II) the experimental points for the various temperatures of the air line up along straight lines representing the relationship $\lg \mathrm{Rb} = f(w - w_p)$. This relationship can be described with sufficient accuracy for practical purposes by the formula

$$\mathrm{Rb} = A \exp[-n(w - w_p)], \quad (14)$$

where $A = 10^{-2}$ and $n = 0.2$ for cloth and $n = 0.83$ for ceramics.

For the peat slab (Fig. 3b) this relationship can be presented in the following manner:

$$\mathrm{Rb}_0 = C(w - w_p)^n. \quad (15)$$

Here the exponent $n = -0.8$ is independent of the air temperature. The coefficient C varies as a function of the drying regime: with a change in the heat-carrier temperature from 100 to 200° C, the coefficient C changes from 0.26 to 0.57. For this case we may assume in approximate terms that

$$C = 0.028 \cdot 10^{-2} t.$$

Somewhat different quantitative relationships are derived for the dimensionless coefficient of thermal diffusivity of the drying process.

The experimental data for the drying of cloth during a period of declining drying rate for various temperatures of the medium (Fig. 4A, curve 1), all other conditions being equal, also line up on a single curve $B = f(\bar{u} - u_p)$; however, the scattering of the experimental points is somewhat greater here than in the generalization for the Rebinder number. Analogous results are obtained for the experimental data in the case in which cloth is being dried at various velocities of heat-carrier motion (Fig. 4A, curve 2)—and the scattering of the experimental points in this case reaches 15–17%.

The effect of the air temperature—in the drying of a ceramic plate—on the magnitude of the dimensionless coefficient of thermal diffusivity B is shown in Fig. 4A (curves 3,4,5).

Thus, the experimental relationship $B = f(\bar{u} - u_p)$ may be approximated by the formula

$$B = k \exp[n(\bar{u} - u_p)]. \quad (16)$$

In this equation for the cloth $k = 2.46$ and $n = -24.8$ for various temperatures of the medium and $k = 1.28$ and $n = -16.7$ for various velocities of air motion. For the ceramic plate $n = -82$ at various temperatures of the heat carrier (the range of investigated temperatures is $t_m = 75$–120° C).

The coefficient k for the ceramic plate is a function of the air temperature and in this temperature range it may be approximated by

$$k = 6.3 \cdot 10^{-3} T_m - 2. \quad (17)$$

Formula (16) is also valid for the drying of a ceramic plate at various values of the relative humidity of the air; however, even here $k = f(\varphi)$, and n is independent of the drying regime.

We see from Fig. 4B that in the drying of a peat slab during a period of declining drying rate for various temperatures, with an air velocity of ($v = 4$ m/sec), and a relative humidity of the air ($\varphi = 5\%$), the experimental points line up in a straight line.

Consequently we will have

$$B = k(w - w_p)^n, \quad (18)$$

where the exponent $n = -0.87$ is independent of the drying regime. The coefficient k in this case is also a linear function of the air temperature

$$k = 4.8 \cdot 10^{-2} T_m - 13.5. \quad (19)$$

The following series of experiments dealt with clarification of the effect of the regime parameters of the drying process on the Rb number and on the coefficient of thermal diffusivity B in the case of a combined radiation-convection influx of heat. In this case a layer of granulated polymers (Capron $d_e = 4$ mm, polystyrene $d_e = 3.4$ mm) was subjected to drying. The experiments were carried out in an installation fitted out with a silicon-carbide heating element in which the emission temperature could be regulated, and so could the velocity of the air moving over the specimens.

The granulated Capron was dried in a layer whose thickness varied (6, 12, and 18 mm). The silicon-carbon heating elements developed temperatures of 723, 823, and 953° K. The moist air ($\varphi = 67\%$) was passed through at speeds of 0.2, 1.5, and 2.0 m/sec. Granulated polystyrene (with an equivalent particle diameter of 3.42 mm) was dried at analogous parameters of T_{rad} (723, 823, 953° K), a relative air humidity $\varphi = 67\%$, and at air velocities of $v = 0.3$, 1.6, and 2.8 m/sec. The thickness of the layer varied (6, 12, 18 mm).

The critical moisture content of the Capron was 4%, while that of the polystyrene was 0.12%. It follows from the temperature curves that the temperature of the Capron rapidly increases with a reduction in the moisture content from 8 to 3% (the initial drying period and a portion of the constant-rate period). Beginning with this moisture content, there is a slight increase in temperature, which corresponds to the nature of the progress in the drying process during the constant-rate period. The temperature of the polystyrene increases continuously with a reduction in the moisture content from 0.4 to 0.002%. Since the critical moisture content of the polystyrene is equal to 0.12% ($w_{cr} = 0.12\%$), the temperature curve encompasses the entire drying period from the initial period to the period of a declining drying rate.

On the basis of the $\bar{t} = f(\bar{u})$ temperature curves we calculated the coefficient of thermal diffusivity for the drying process:

$$B = \frac{d\bar{t}}{d\bar{u}} \frac{\bar{u}_{cr}}{T_\infty}, \quad (20)$$

where for the constant parameters $\Delta\bar{u}$ and ΔT we assumed: $\Delta\bar{u} = \bar{u}_{cr}$ (the critical moisture content, in kg/kg) and $\Delta T = T_\infty$.

The temperature T_∞ was calculated from the magnitude of the air temperature T_m, the density of the radiant flux q_{rad}, and from the heat-transfer coefficient α

$$T_\infty = T_m + \frac{q_{rad}}{\alpha}. \quad (21)$$

The results of the calculations are shown in Fig. 5; here it was established that the coefficient B is a uniquely defined function of the moisture content and that it is independent of the regime parameters T_{rad}, T_m, φ, and v within the limits cited earlier. We see from Fig. 5 that the experimental points are situated along straight lines. Consequently, we can write the following empirical formulas:

for polystyrene

$$B = 0.0116\bar{u}^{-0.18}, \qquad (22)$$

for Capron

$$B = 7.95\bar{u}^{1.6}. \qquad (23)$$

The positive exponent n = 1.6 for the Capron is explained by the fact that formula (23) corresponds primarily to the initial drying period in which the variation in the $\bar{t} = f(\bar{u})$ temperature curve is different from the period of a declining rate.

The Rebinder number (Rb = br/c) was calculated in accordance with the data of the coefficient of thermal diffusivity ($b = d\bar{t}/d\bar{u}$) of the drying process. The results of the calculations are shown in Fig. 6. The experimental points exhibit considerable scatter. This is explained primarily by the fact that we lack sufficiently accurate data on the heat of moisture evaporation r which consists of the heat of liquid evaporation r_l and the heat of wetting r_w. The latter is virtually unknown for the given materials. Moreover, the specific heat capacity of the material may vary as a result of the moisture content in accordance with a law that it is not linear. Thus only in the very roughest first approximation can we describe the experimental data presented in Fig. 6 in the form of empirical formulas:

for polystyrene

$$\mathrm{Rb} = 0.025\bar{u}^{-0.48}, \qquad (24)$$

for Capron

$$\mathrm{Rb} = 0.456\cdot 10^{-3}\bar{u}^{1.7}. \qquad (25)$$

The function $B = f(\bar{u})$ makes it possible to determine the temperature of the material during any instant of the drying process, while the empirical formulas $\mathrm{Rb} = f(\bar{u})$ make it possible to calculate the intensity of heat transfer throughout the entire drying process, thus avoiding the necessity of determining the coefficient of heat transfer. The drying rate was calculated from conventional approximate relationships.

NOTATION

a is the heat diffusion coefficient; a_m is the diffusion coefficient for the moisture content in the body; b is the thermal diffusivity coefficient; B is the relative thermal diffusivity coefficient; c is the specific heat capacity of a moist body; c_l is the specific heat capacity of a liquid; c_0 is the specific heat capacity of an absolute dry body; j is the moisture transfer intensity; K is the drying coefficient; N is the drying rate in the first period; q is the heat transfer intensity; R_V is the ratio of body volume to its surface (characteristic size); r is the specific heat of evaporation, $\bar{t}$ is the mean body temperature; t_m is the temperature of the medium; T is the absolute temperature (T = t + 273° K); $\bar{u}$ is the mean (with respect to volume) moisture content of the body; $\bar{u}_0$ is the initial moisture content; v is the velocity of air motion; w is the moisture content in percent (w = 100u); ρ_0 is the density of an absolute dry body; τ is the time; φ is the relative air humidity; $\varkappa$ is the relative drying coefficient; Rb is the Rebinder number; Ko is the Kossovich criterion; Lu is the criterion of moisture-heat transfer; $Ki_q(\tau)$ and $Ki_m(\tau)$ are the heat and moisture transfer Kirpichev numbers, respectively. Subscripts: cr is the critical value; 0 is the state of an absolute dry body; c is the period of constant rate; m is the state of the medium; rad is the parameter of the generator;* is the dimensionless relative quantity.

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